1 Set Operations

- $\mathbb{R}$, the set of real numbers

- $\mathbb{Q}$, the set of rational numbers: $\{a/b : a, b \in \mathbb{Z} \wedge b \neq 0\}$

- $\mathbb{Z}$, the set of integers: $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$

- $\mathbb{N}$, the set of natural numbers: $\{0, 1, 2, 3, \ldots\}$

(a) Given a set $A = \{1, 2, 3, 4\}$, what is $\mathscr{P}(A)$ (Power Set)?

(b) Given a generic set B, how do you describe $\mathscr{P}(B)$ using set comprehension notation? (Set Comprehension is $\{x \mid x \in A\}$.)

(c) What is $\mathbb{R} \cap \mathscr{P}(A)$?

(d) What is $\mathbb{R} \cap \mathbb{Z}$?

(e) What is $\mathbb{N} \cup \mathbb{Q}$?

(f) What kind of numbers are in $\mathbb{R} \setminus \mathbb{Q}$?

(g) If $S \subseteq T$, what is $S \setminus T$?

2 Image and Preimage

Let X and Y be sets, and $f : X \to Y$ be a function. For a subset, $A \subseteq X$, define it's image to be $f(A) = \{f(x) \mid x \in A\}$. For a subset $B \subseteq Y$, define it's preimage $f^{-1}(B) = \{x \mid f(x) \in B\}$. Note that in this context f^{-1} does not refer to an inverse function, as f may not have an inverse.

(a) Let $B \subseteq F(X)$. Prove that $f(f^{-1}(B)) = B$

(b) Let $A \subseteq X$. Prove that $A \subseteq f^{-1}(f(A))$

(c) Give an example of when $A \neq f^{-1}(f(A))$

(d) Suppose f is injective. Is it true that $A = f^{-1}(f(A))$? Prove or provide a counter-example.

3 Bijections

Consider the function

$$f(x) = \begin{cases} x, & \text{if } x \geq 1; \\ x^2, & \text{if } -1 \leq x < 1; \\ 2x+3, & \text{if } x < -1. \end{cases}$$

(a) If the domain and range of f are $\mathbb{N}$, is f injective (one-to-one), surjective (onto), bijective?

(b) If the domain and range of f are $\mathbb{Z}$, is f injective (one-to-one), surjective (onto), bijective?

(c) If the domain and range of f are $\mathbb{R}$, is f injective (one-to-one), surjective (onto), bijective?