CS 70 Discrete Mathematics and Probability Theory
Fall 2012 Vazirani **Midterm 2**

1 Counting

For each question write the answer as an expression involving factorials, powers and binomial coefficients, and a one sentence explanation:

1. The number of ways of choosing 6 cards from a standard deck of cards to get three pairs.

2. The number of bridge hands with a $(6, 3, 2, 2)$ distribution. i.e. 6 cards in one suite, 3 in another, and 2 each in the remaining suites.

3. The number of anagrams of CANDIDATE.

4. The number of rolls of 6 dice with exactly 4 distinct values.

5. The number of (strictly) increasing sequences of k numbers from $\{1, 2, \ldots, n\}$. e.g. the sequence $1, 3, 4, 7, 20$ is (strictly) increasing.

6. The number of integers $x: \ 0 \leq x < 41$ such that $4x = 21 (\mathrm{mod} 41)$.

7. The number of seven digit numbers where the first digit is not 0 and no pair of adjacent digits are the same.

2 Calculating Probabilities

The Tigers are down 1-2 in the World series (i.e. the Tigers have won 1 and the Giants 2 of the first 3 games out of a best of seven games series). The Tigers are still think they stand a good chance of winning the series since they estimate that their chances of winning any given game is 0.6 (the outcomes of different games are independent).

1. If you believe their assumption, what is the probability that the Tigers will win the series?

2. The assistant coach of the Tigers believes that the situation is quite dire, and the Tigers should pull out all stops in Game 4. He reckons that if they put in a super human effort they can improve their chances of winning to with 0.75. The coach points out that if they do so, they will be so mentally spent that their chances of winning in subsequent games will drop to 0.5. What is their chance of winning the tournament under this strategy?

   Remember you may leave your answers as expressions involving factorials, etc.

3 Lines

For this question, assuming that you are working modulo p, where p is a prime greater than 10. Select a random line (a polynomial $A(x)$ of degree at most 1).

1. What is the chance that it goes through a particular point, (x, y), for example if $(x, y) = (0, 5)$, the question asks what is the probability that $A(0) = 5$?

2. What is the chance that it goes through two particular points $(x_1, y_1),\ (x_2, y_2)$, where $x_1 \neq x_2$?

3. What about 3 particular points $(x_1, y_1),\ (x_2, y_2),\ (x_3, y_3)$, where $x_1,\ x_2,\ x_3$ are distinct?

4 Graphs:

A *c*omplete bipartite graph is an undirected graph whose vertex set is the union of two disjoint non-empty sets, V_0 and V_1, and whose edges consist of all pairs $\{u, v\} : \ u \in V_0$ and $v \in V_1$. (there are no edges between pairs of vertices in V_0 or pairs of vertices in V_1)

1. Prove that a complete bipartite graph is connected. i.e. there is a path between every pair of vertices.

2. Prove that for any graph, G, either G or $\bar{G}$ (the complement of G) is connected. Here $\bar{G}$ has the same vertex set as G, but a pair of vertices is adjacent in $\bar{G}$ iff there is no edge between them in G.

5 Democratic convention

At the democratic convention, President Obama is scheduled to give his acceptance speech at an outdoor stadium. In recent years, on average it has rained on only 10% of the evenings in September. Unfortunately, the weatherman has predicted rain for the evening of the speech. Going through the historical records, on evenings when it rained, the weatherman's predictions were correct 80% of the time. And on evenings when it did not rain he incorrectly forecast rain 20% of the time. What is the probability that it will rain on the evening of President Obama's acceptance speech? Write your answer as a rational number, i.e. in the form $\frac{a}{b}$.