

1 Warm-up

For each of the following parts, you may leave your answer as an expression.

- (a) You throw darts at a board until you hit the center area. Assume that the throws are i.i.d. and the probability of hitting the center area is $p = 0.17$. What is the probability that you hit the center on your eighth throw?
- (b) Let $X \sim \text{Geometric}(0.2)$. Calculate the expectation and variance of X .
- (c) Suppose the accidents occurring weekly on a particular stretch of a highway is Poisson distributed with average number of accidents equal to 3 cars per week. Calculate the probability that there is at least one accident this week.
- (d) Consider an experiment that consists of counting the number of α particles given off in a one-second interval by one gram of radioactive material. If we know from past experience that, on average, 3.2 such α -particles are given off per second, what is a good approximation to the probability that no more than 2 α -particles will appear in a second?

2 Coupon Collector Variance

It's that time of the year again - Safeway is offering its Monopoly Card promotion. Each time you visit Safeway, you are given one of n different Monopoly Cards with equal probability. You need to collect them all to redeem the grand prize.

Let X be the number of visits you have to make before you can redeem the grand prize. Show that $\text{Var}(X) = n^2 \left(\sum_{i=1}^n i^{-2} \right) - \mathbb{E}(X)$. *[Hint: Try to express the number of visits as a sum of geometric random variables as with the coupon collector's problem. Are the variables independent?]*

3 Boutique Store

Consider a boutique store in a busy shopping mall. Every hour, a large number of people visit the mall, and each independently enters the boutique store with some small probability. The store owner decides to model X , the number of customers that enter her store during a particular hour, as a Poisson random variable with mean λ .

Suppose that whenever a customer enters the boutique store, they leave the shop without buying anything with probability p . Assume that customers act independently, i.e. you can assume that they each flip a biased coin to decide whether to buy anything at all. Let us denote the number of customers that buy something as Y and the number of them that do not buy anything as Z (so $X = Y + Z$).

- (a) What is the probability that $Y = k$ for a given k ? How about $\mathbb{P}[Z = k]$? *Hint*: You can use the identity

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}.$$

- (b) State the name and parameters of the distribution of Y and Z .
- (c) Prove that Y and Z are independent. In particular, prove that for every pair of values y, z , we have $\mathbb{P}[Y = y, Z = z] = \mathbb{P}[Y = y]\mathbb{P}[Z = z]$.