

1 Condition on an Event

The random variable X has the PDF

$$f_X(x) = \begin{cases} cx^{-2}, & \text{if } 1 \leq x \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Determine the value of c .
- (b) Let A be the event $\{X > 1.5\}$. Calculate $\mathbb{P}(A)$ and the conditional PDF of X given that A has occurred.

2 Max of Uniforms

Let X_1, \dots, X_n be independent $U[0, 1]$ random variables, and let $X = \max(X_1, \dots, X_n)$. Compute each of the following in terms of n .

- (a) What is the cdf of X ?
- (b) What is the pdf of X ?
- (c) What is $\mathbb{E}[X]$?
- (d) What is $\text{Var}[X]$?

3 Darts but with ML

Suppose Alice and Bob are playing darts on a circular board with radius 1. When Alice throws a dart, the distance of the dart from the center is uniform $[0, 1]$. When Bob throws the dart, the location of the dart is uniform over the whole board. Let X be a random variable corresponding to the distance of the player's dart from the center.

- (a) What is the pdf of X if Alice throws
- (b) What is the pdf of X if Bob throws
- (c) Suppose we let Alice throw the dart with probability p , and let Bob throw otherwise. What is the pdf of X (your answer should be in terms of p)?
- (d) Using the same premise as in part c, suppose you observe a dart on the board but don't know who threw it. Let x be the dart's distance from the center. We would like to come up with a decision rule to determine whether Alice or Bob is more likely to have thrown the dart given your observation, x . Specifically, if we let A be the event that Alice threw the dart and B be the event that Bob threw, we want to guess A if $\mathbb{P}[A|X \in [x, x + dx]] > \mathbb{P}[B|X \in [x, x + dx]]$ (what do these two probabilities have to sum up to?). For what values of x would we guess A ? (your answer should be in terms of p)