CS 70 Discrete Mathematics and Probability Theory
Summer 2020 Course Notes

DIS 6D

1 Continuous Joint Densities

The joint probability density function of two random variables X and Y is given by $f(x,y) = Cxy$ for $0 \leq x \leq 1, 0 \leq y \leq 2$, and 0 otherwise (for a constant C).

(a) Find the constant C that ensures that $f(x,y)$ is indeed a probability density function.

(b) Find $f_X(x)$, the marginal distribution of X.

(c) Find the conditional distribution of Y given $X = x$.

(d) Are X and Y independent?

2 Uniform Distribution

You have two spinning wheels, each having a circumference of 10 cm with values in the range $[0, 10)$ marked on the circumference. If you spin both (independently) and let X be the position of the first spinning wheel's mark and Y be the position of the second spinning wheel's mark, what is the probability that $X \geq 5$, given that $Y \geq X$?

3 Exponential Practice

Let $X \sim \text{Exponential}(\lambda_X)$ and $Y \sim \text{Exponential}(\lambda_Y)$ be independent, where $\lambda_X, \lambda_Y > 0$. Let $U = \min\{X,Y\}$, $V = \max\{X,Y\}$, and $W = V - U$.

(a) Compute $\mathbb{P}(U > t, X \leq Y)$, for $t \geq 0$.

(b) Use the previous part to compute $\mathbb{P}(X \leq Y)$. Conclude that the events $\{U > t\}$ and $\{X \leq Y\}$ are independent.

(c) Compute $\mathbb{P}(W > t \mid X \leq Y)$.

(d) Use the previous part to compute $\mathbb{P}(W > t)$.

(e) Calculate $\mathbb{P}(U > u, W > w)$, for $w > u > 0$. Conclude that U and W are independent. [*Hint*: Think about the approach you used for the previous parts.]